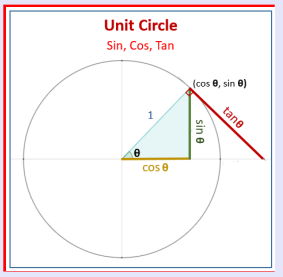
Trigonometry

Lecture 33



Feb 19-8:47 AM

Verify $\dfrac{2}{\tan x + \cot x} = \sin 2x$

$$\frac{2}{\tan x + \cot x} = \frac{2}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{2\sin x \cos x}{\sin^2 x + \cos^2 x}$$

LCD $= \cos x \sin x$

$$= \frac{\sin 2x}{1}$$

$$= \sin 2x$$

Oct 28-10:35 AM

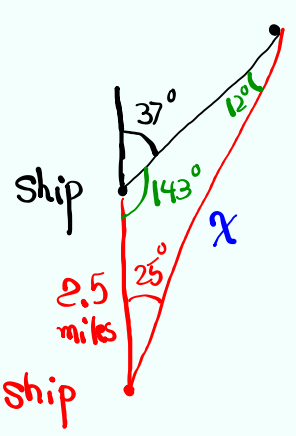
Simplify $\dfrac{\cos^2 t}{1-\sin t} - \dfrac{1}{\csc t}$

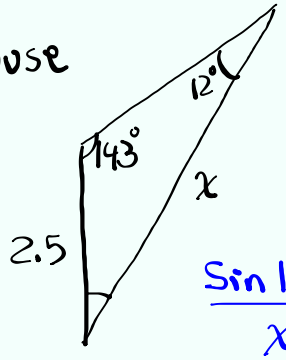
$= \dfrac{1-\sin^2 t}{1-\sin t} - \sin t$

$= \dfrac{(1+\sin t)(1-\sin t)}{1-\sin t} - \sin t$

$= 1 + \sin t - \sin t = \boxed{1}$

Oct 28-10:38 AM





$\dfrac{\sin 143^\circ}{x} = \dfrac{\sin 12^\circ}{2.5}$

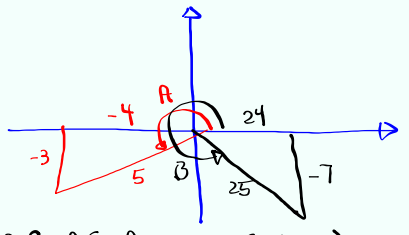
$x \sin 12^\circ = 2.5 \sin 143^\circ$

$x = \dfrac{2.5 \sin 143^\circ}{\sin 12^\circ}$

$\boxed{x \approx 7.2 \text{ miles}}$

Oct 28-10:41 AM

$\sin A = -\frac{3}{5}$ A is in QIII , $\cos B = \frac{24}{25}$ B is in QIV



$\sin 2A = 2 \sin A \cos A$

$= 2 \cdot \frac{-3}{5} \cdot \frac{-4}{5} = \frac{24}{25}$

$\cos(A-B) =$

$\cos A \cos B + \sin A \sin B =$

$\frac{-4}{5} \cdot \frac{24}{25} + \frac{-3}{5} \cdot \frac{-7}{25} =$

$\frac{-96+21}{125} = \frac{-75}{125}$

$= \boxed{\frac{-3}{5}}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

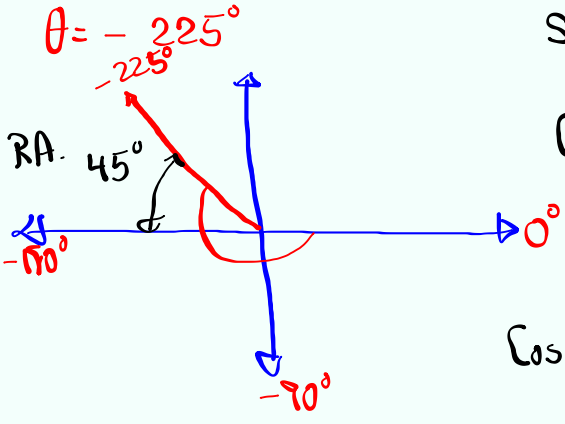
$= \frac{\frac{3}{4} + \frac{-7}{24}}{1 - \frac{3}{4} \cdot \frac{-7}{24}}$

LCD $= 4 \cdot 24 = 96$

$= \frac{3 \cdot 24 + (-7)(4)}{96 + 21} = \frac{72 - 28}{117} = \boxed{\frac{44}{117}}$

Oct 28-10:45 AM

$\theta = -225°$



$\sin(-225°) = +\sin 45° = \frac{\sqrt{2}}{2}$

$\csc(-225°) = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \sqrt{2}$

$\cos(-225°) = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\sec(-225°) = -\sqrt{2}$

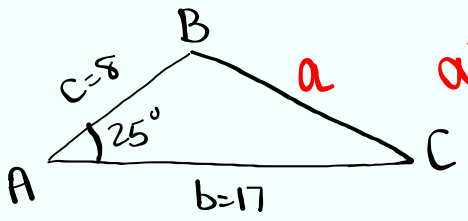
$\tan(-225°) = -\tan 45° = -1$

$\cot(-225°) = -\cot 45° = -1$

Oct 28-10:53 AM

Find a if $A = 25°$, $b = 17$ ft, $c = 8$ ft

Law of Cosines



$a^2 = b^2 + c^2 - 2bc\cos A$

$= 17^2 + 8^2 - 2 \cdot 8 \cdot 17 \cdot \cos 25°$

$= 106.484$

$a \approx 10$ ft

Oct 28-10:58 AM

Graph $y = 2\sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

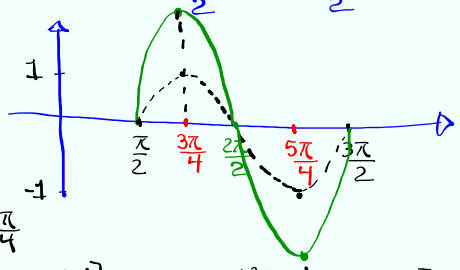
Amplitude

$0 \le 2\left(x - \frac{\pi}{2}\right) \le 2\pi$

$0 \le 2x - \pi \le 2\pi$

$\pi \le 2x \le 3\pi$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$



Test Point $\frac{3\pi}{4}$

$y = 2\sin\left[2\left(\frac{3\pi}{4} - \frac{\pi}{2}\right)\right] = 2\sin 2\left(\frac{3\pi - 2\pi}{4}\right) = 2\sin 2 \cdot \frac{\pi}{4}$

$= 2\sin\frac{\pi}{2}$

$= 2 \cdot 1 = \boxed{2}$

Oct 28-11:01 AM

Graph $y = -2 + \cos\left(\frac{1}{2}(x+\frac{\pi}{4})\right)$

Vertical shift 2 down

$0 \le \frac{1}{2}(x+\frac{\pi}{4}) \le 2\pi$

$2 \cdot 0 \le x + \frac{\pi}{4} \le 2 \cdot 2\pi$

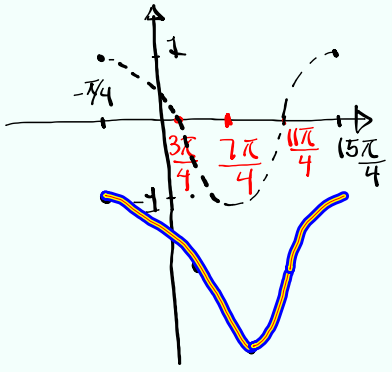
$0 \le x + \frac{\pi}{4} \le 4\pi$

$4 \cdot 0 \le 4x + 4 \cdot \frac{\pi}{4} \le 4 \cdot 4\pi$

$0 \le 4x + \pi \le 16\pi$

$-\pi \le 4x \le 15\pi$

$-\frac{\pi}{4} \le x \le \frac{15\pi}{4}$



Oct 28-11:08 AM

Graph $y = \tan\left(\frac{1}{4}\pi x\right)$

$-\frac{\pi}{2} < \frac{1}{4}\pi x < \frac{\pi}{2}$

LCD = 4

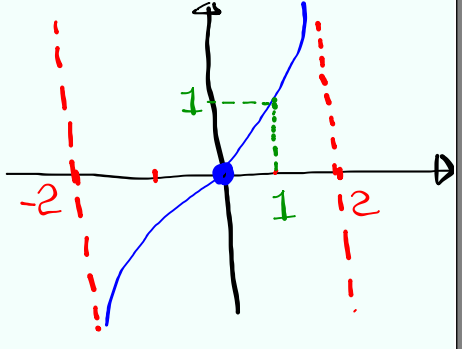
$-2\pi < \pi x < 2\pi$

Divide by π

$-2 < x < 2$

$y = \tan\left(\frac{\pi}{4} \cdot 1\right)$

$= \tan\frac{\pi}{4} = 1$



Oct 28-11:16 AM

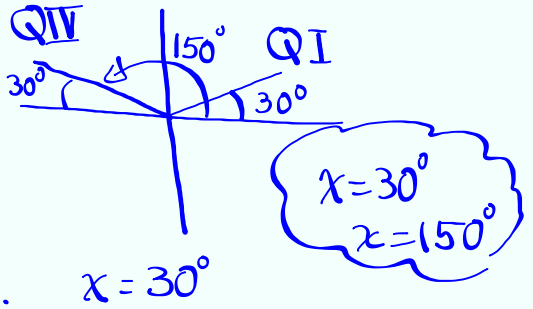
Intro to Trig. Equations:

Solve $2\sin x - 1 = 0$ on $0° \le x < 360°$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

R.A. $30°$



QI $x = \text{R.A.}$ $x = 30°$

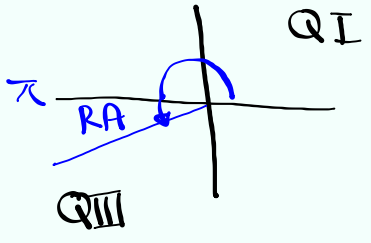
QII $x = 180° - \text{RA.} = 180° - 30° = 150°$

Oct 28-11:20 AM

Solve $\tan x - 1 = 0$ $0 \le x < 2\pi$

$\tan x = 1$

RA. $45° = \frac{\pi}{4}$



QI $x = \text{R.A.}$ $x = \frac{\pi}{4}$

QIII $x = \pi + \text{RA}$ $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

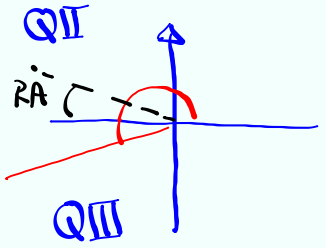
$\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

Oct 28-11:25 AM

Solve $2\cos x + \sqrt{3} = 0$ → $0° \le x < 360°$

$\cos x = -\frac{\sqrt{3}}{2}$

R.A. $30°$



QII $x = 180° - \text{R.A.}$ $x = 180° - 30° = 150°$

QIII $x = 180° + \text{RA}$ $x = 180° + 30° = 210°$

Oct 28-11:28 AM

Solve $\tan^2 x - 3 = 0$ for $0 \le x < 2\pi$

Hint: Isolate $\tan x$.

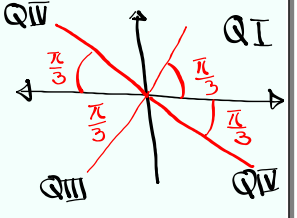
$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \sqrt{3}$ QI & QIII

$\tan x = -\sqrt{3}$ QII & QIV

R.A. $\frac{\pi}{3}$



QI $x = \text{R.A.} = \frac{\pi}{3}$

QII $x = \pi - \text{RA} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

QIII $x = \pi + \text{RA} = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

QIV $x = 2\pi - \text{R.A.} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Oct 28-11:32 AM

Solve $2\sin^2 x - \sin x - 1 = 0$ for

$0° \le x < 360°$

Hint: Factor LHS

$(2\sin x + 1)(\sin x - 1) = 0$

+sinx

−2sinx

Zero-Product Rule

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (maybe both)

$2\sin x + 1 = 0$ OR $\sin x - 1 = 0$

Oct 28-11:39 AM